EEL3701

Menu

- Minterms & Maxterms
- SOP & POS
- MSOP & MPOS
- Simplification using the theorems/laws/axioms

1

EEL3701

Algebraic Simplification - Boolean Algebra

- Definitions (Review)

Minterms (written as m_i): A conjunctive (AND) term that is 1 in one and one only row of an exhaustive truth table.

Analogy: A minterm is like a column unit vector, u_i, in 2^n space, (where n = number of Boolean Variables), e.g., $m_2=[0, 0, 1, 0]^T$ (only one 1)

Maxterms (written as M_i): A disjunctive (OR) term that is 0 in one and one only row of an exhaustive truth table.

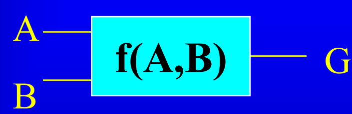
Analogy: A maxterm is like a complement of a unit vector u_i in 2^n space, e.g., $M_2=[1, 1, 0, 1]^T$ (i.e., only one 0)

Definition: Any function can be written as a *product of sums* that is as a conjunction of disjunctive terms (AND of ORs).

2

EEL3701 Algebraic Simplification - Boolean Algebra

- Example : Suppose G = f(A,B)



m_i = minterm

M_i = maxterm

A	B	m_3	m_2	m_1	m_0	M_3	M_2	M_1	M_0
0	0	0	0	0	1	1	1	1	0
0	1	0	0	1	0	1	1	0	1
1	0	0	1	0	0	1	0	1	1
1	1	1	0	0	0	0	1	1	1

NOTE: $M_0 = /m_0$ i.e., $M_i = /m_i$

University of Florida, EEL 3701 – File **08**
© Drs. Schwartz & Arroyo

3

3

EEL3701 POS & SOP Forms

- Suppose G = *True in 0th & 2nd rows*

		M_2	m_2	M_0	m_0			
A	B	$/m_2$	$/M_2$	$/m_0$	$/M_0$	G	M_3	M_1
0	0	1	0	0	1	1	1	1
0	1	1	0	1	0	0	1	0
1	0	0	1	1	0	1	1	1
1	1	1	0	1	0	0	0	1

$m_i = /M_i$

- There are two possible ways of representing G:
  - a) Look for 1's: SOP (Sum Of Products; an OR of minterms) form
  - b) Look for 0's: POS (Product of Sums; an AND of maxterms) form

University of Florida, EEL 3701 – File **08**
© Drs. Schwartz & Arroyo

4

4

EEL3701 POS & SOP Solutions

a) Look for 1's: *SOP (Sum Of Products) Form*

Obviously, we can synthesize G as an OR of minterms, i.e., a sum of products (conjunctive [AND] terms)

$$\therefore G = m_0 + m_2 = m_0 \vee m_2 = \overline{A}\bullet\overline{B} + A\bullet\overline{B} = (\overline{A}\wedge\overline{B})\vee(A\wedge\overline{B})$$

* We'll use the •, + notation:

$$\therefore G = \overline{A}\bullet\overline{B} + A\bullet\overline{B} \equiv G_{SOP}$$

		M_2	m_2	M_0	m_0			
A	B	$/m_2$	$/M_2$	$/m_0$	$/M_0$	G	M_3	M_1
0	0	1	0	0	1	1	1	1
0	1	1	0	1	0	0	1	0
1	0	0	1	1	0	1	1	1
1	1	1	0	1	0	0	0	1

b) Look for 0's: *POS (Product of Sums) Form*

Not so obvious is the fact that G can also be synthesized as an AND of maxterms, i.e., a product of sums (disjunctive terms; OR terms)

$$\therefore G = M_1 \bullet M_3 = (A + \overline{B})\bullet(\overline{A} + \overline{B}) \equiv G_{POS}$$

NOTE: It must be true that $G = G_{SOP} = G_{POS}$

5

EEL3701 Algebraic Simplification - Boolean Algebra

- We could have synthesized the function f (from lecture 6) using the 0's, as a POS (instead of with a SOP)
  - >[RECALL]: Maxterms (ORs of literals) are places (columns) in an exhaustive truth table that are 1 everywhere except in one and only one row
  - >Therefore, to obtain the zeros for f, we take products (ANDs) of maxterms
- Since any function f can be synthesized as a product (conjunction) of sums (ORs; disjunctions), i.e., a POS, then

$$f = \prod_i \text{Maxterm}_i \quad \text{ALSO} \quad f = \sum_i \text{Minterm}_i$$

6

EEL3701 POS (Product of Sums)

- Ex: Synthesize the function f (from lecture 7: Boolean Algebra)

a	b	c	M_7	M_6	M_5	M_4	M_3	M_2	M_1	M_0	f(a,b,c)
0	0	0	1	1	1	1	1	1	1	0	1
0	0	1	1	1	1	1	1	1	0	1	0
0	1	0	1	1	1	1	1	0	1	1	1
0	1	1	1	1	1	1	0	1	1	1	1
1	0	0	1	1	1	0	1	1	1	1	0
1	0	1	1	1	0	1	1	1	1	1	1
1	1	0	1	0	1	1	1	1	1	1	1
1	1	1	0	1	1	1	1	1	1	1	0

$f_{SOP} = m_0 + m_2 + m_3 + m_5 + m_6$ = /a /b /c + /a b /c + /a b c + a /b c + a b /c

$f_{POS} = M_1 \bullet M_4 \bullet M_7$ = (a + b + /c) • (/a + b + c) • (/a + /b + /c)

EEL3701 POS versus SOP

f_{SOP} = /a/b/c + /a b/c + /a b c + a/b c + a b/c

- Ex (continued): f has 0's in rows 1, 4, and 7, i.e., when (a,b,c) are (0,0,1), (1,0,0), and (1,1,1), respectively, then
  - > M_1 = a + b + /c M_4 = /a + b + c M_7 = /a +/b + /c

  $f_{POS} = M_7 \bullet M_4 \bullet M_1$ = (/a + /b + /c) • (/a + b + c) • (a + b + /c)
- See how dissimilar the POS & SOP are? But they represent the same f !!! They must be equivalent expressions.

  f_{POS} = (/a + /b + /c) • (/a + b + c) • (a + b + /c)

  = /a /b /c + /a b /c + /a b c + a /b c + a b /c = f_{SOP}
- The f_{SOP} was simplified. Can the f_{POS} be simplified? No
- Is it possible that even after simplifying, f_{SOP} may require fewer gates than f_{POS} or vice versa? Yes

a	b	c	M_7	M_6	M_5	M_4	M_3	M_2	M_1	M_0	f(a,b,c)
0	0	0	1	1	1	1	1	1	1	0	1
0	0	1	1	1	1	1	1	1	0	1	0
0	1	0	1	1	1	1	1	0	1	1	1
0	1	1	1	1	1	1	0	1	1	1	1
1	0	0	1	1	1	0	1	1	1	1	0
1	0	1	1	1	0	1	1	1	1	1	1
1	1	0	1	0	1	1	1	1	1	1	1
1	1	1	0	1	1	1	1	1	1	1	0

EEL3701 Minimum Product of Sums (MPOS)

- The POS can also be simplified by applying the theorems of Boolean Algebra
- When you cannot reduce it further, the resulting expression is called a *minimum product of sums* or an MPOS for short.
- In our example, we obtained

$$f_{MSOP} = /a\ b + /a\ /c + a\ /b\ c + b\ /c$$
$$f_{MPOS} = (/a + /b + /c) \bullet (/a + b + c) \bullet (a + b + /c)$$

- It must be true that $f = f_{POS} = f_{SOP}$ & that $f = f_{MSOP} = f_{MPOS}$. However, it may be that the f_{MSOP} may require fewer gates than f_{MPOS}, or vice versa.
- How many gates do each take? **Do it!**

EEL3701 Simplification: Duality

- Useful **Simplification Theorems**:

	Dual	
XY + X /Y = X	⟺	(X+Y)(X + /Y) = X
X + XY = X		X(X+Y) = X
(X + /Y)Y = XY		X /Y + Y = X + Y

  - These are all special cases of the distributive law

    X(Y + Z) = XY + XZ ⟺ (Dual) X + YZ = (X +Y)(X + Z)

- Important Rules
  - 8. Distributive Laws — 9-11. Simplification Thms
  - 12-13. DeMorgan's Laws — 14-15. Duality
  - 16. Theorem for multiplying out and factoring
  - 17. Consensus Theorem

EEL3701 Simplification: Consensus Theorem

- Consensus Theorem: XY + **YZ** + /X Z = XY + /X Z

YZ is called the consensus of these terms. Therefore, YZ can be eliminated. (Proved by Kline, Harvard, ~1940)

- Proof
  - >Since X+/X=1 and 1•YZ=YZ, then
  - >f = XY + YZ + /X Z = XY + (X+/X)YZ + /X Z
  - > = XY + XYZ + /X YZ + /X Z
  - >After rearranging terms, we get
  - >f = XY + XYZ + /X Z + /X YZ
  - > By distribution: f = XY(1+Z) + /X Z(1+Y)
  - >Since 1+A=1 and 1•A=A, we get
  - >f= XY(1+Z) + /X Z(1+Y)= XY•1 + /X Z•1 = **XY + /X Z**

QED

11

EEL3701 Simplification: Duality Example

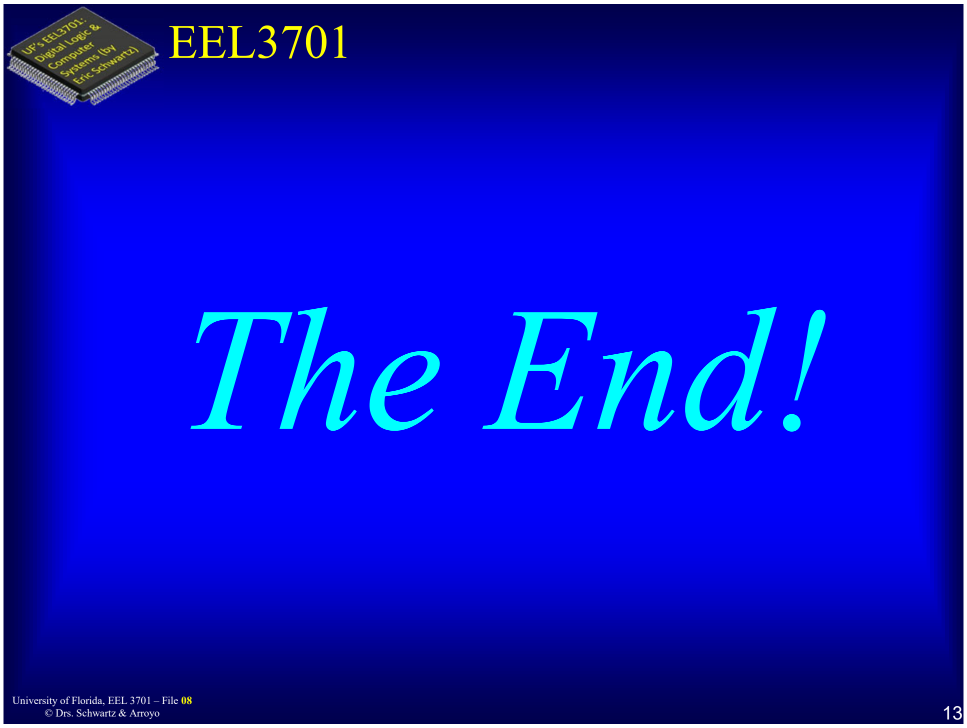
- Duality can help a lot!
- Ex 1:
  - > (/A+B) • (/A+/B) =?= **/A** □
  - > (/A•B) + (/A•/B)= **/A(B+/B) = /A**

DUAL

- Ex 2:
  - > (A+B+C) • (A+B+/C) =?= **A+B** □
  - > (A•B•C) + (A•B•/C) = **A•B(C+/C) = A•B**

DUAL

12


UF's EEL3701: Digital Logic & Computer Systems (by Eric Schwartz)
EEL3701
The End!
University of Florida, EEL 3701 – File 08
© Drs. Schwartz & Arroyo
13

13